Assessment meets Learning: On the relation between Item Response Theory and Bayesian Knowledge Tracing

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ACTNext

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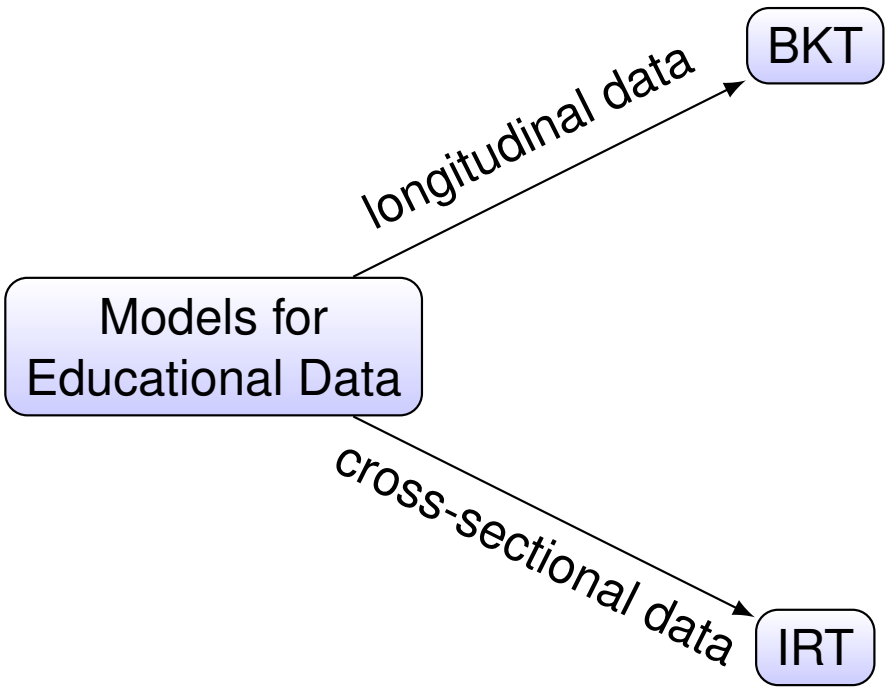
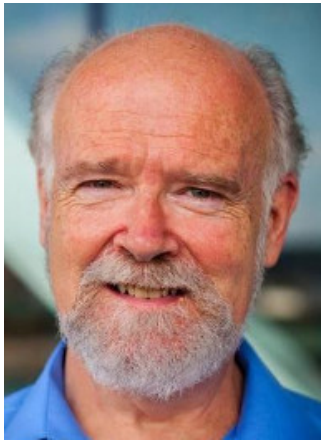
Introduction

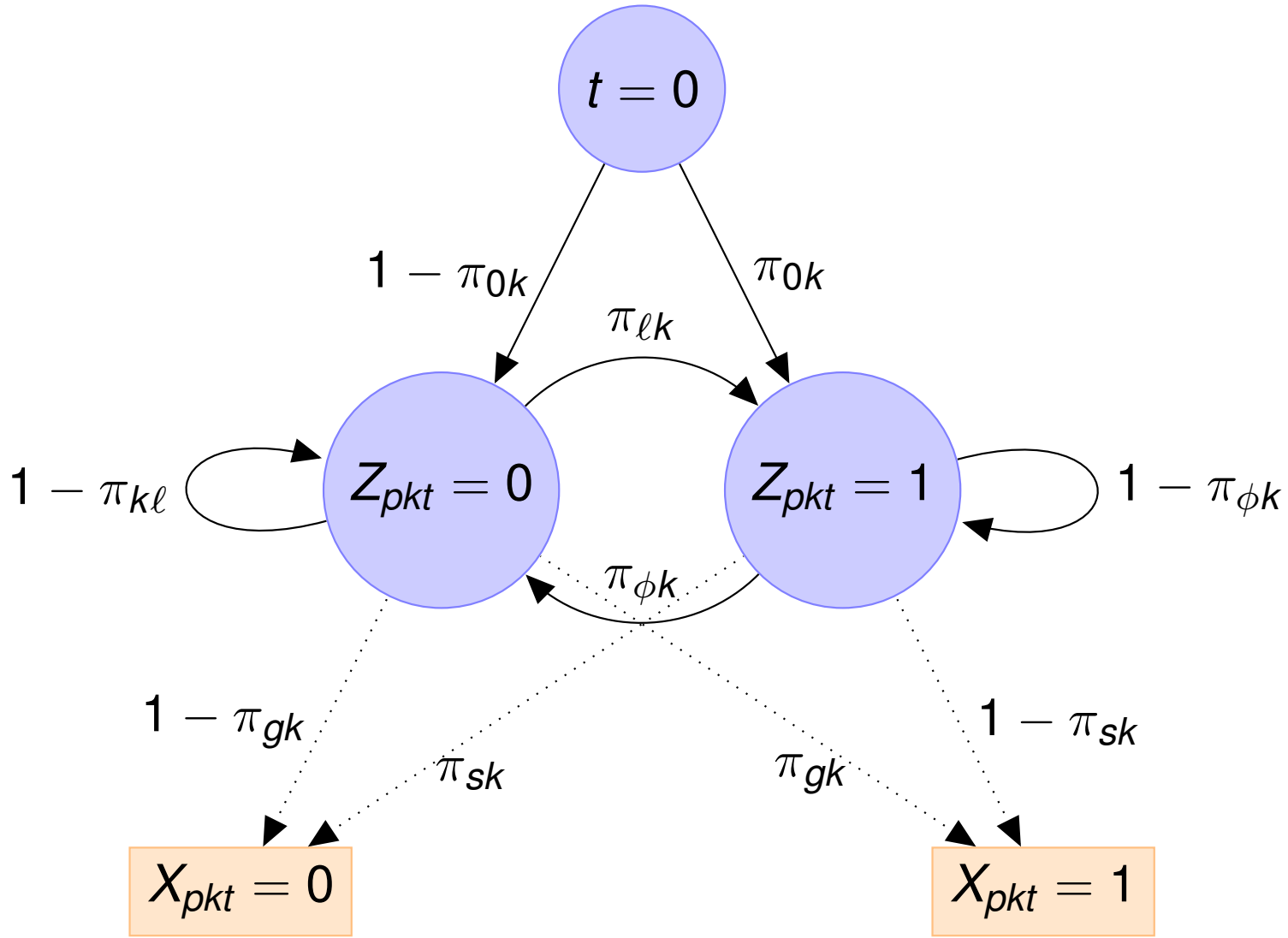




Bayesian Knowledge Tracing

- Standard model used for learning data from intelligent tutoring systems (Corbett and Anderson 1994).
- Hidden Markov model with binary hidden/latent state and binary emission distribution
- Standard BKT model utilizes five global (i.e. shared among all individuals) parameters per skill and treats skills independently

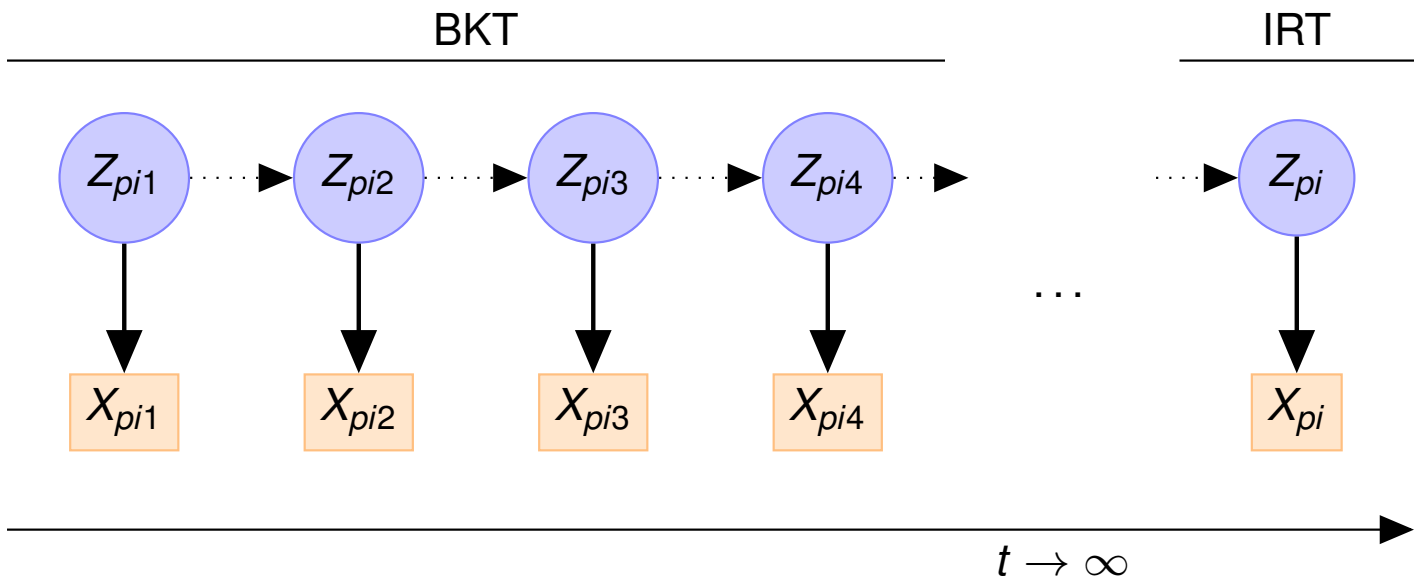
Bayesian Knowledge Tracing



Equilibrium Distribution of Latent Mastery

- We construct a specific BKT model with person specific learn rate, $\pi_{\ell p}$, item specific forget rate, $\pi_{\phi i}$, and item specific slip and guess parameters π_{si} and π_{gi}.
- Let X_{pit} be the response of person p to item i at time t.
- Let Z_{pit} be the latent mastery of person p for item i at time t.

Equilibrium Distribution of Latent Mastery



- We show that the equilibrium distribution of the latent mastery variable in BKT follows a particular parameterization of an IRT model.
- Let Z_{pi} be the distribution of Z_{pit} at equilibrium (as $t \to \infty$). We show

$$\Pr(Z_{pi} = 1) = \frac{\pi_{\ell p}}{\pi_{\ell p} + \pi_{\phi i}}$$

Equilibrium Distribution of Latent Mastery

- Let the transition matrix of the BKT model be denoted by $\mathbf{A}_{pi}$

$$\mathbf{A}_{pi} = \begin{pmatrix} 1 - \pi_{\phi i} & \pi_{\phi i} \\ \pi_{\ell p} & 1 - \pi_{\ell p} \end{pmatrix}$$

- Let $\left\{ Z_{pit} \right\}_{t=1,\ldots,T_p}$ denote the Markov chain formed by the transitions of the latent state variable in the BKT model.
- The stationary distribution for this Markov chain, $\boldsymbol{\lambda}^{\intercal} = (\lambda_0, \lambda_1)$, exists and is the unique solution which satisfies $\boldsymbol{\lambda} = \mathbf{A}_{pi}^{\intercal} \boldsymbol{\lambda}$ where $\lambda_0 = P(Z_{pi} = 0)$, $\lambda_1 = P(Z_{pi} = 1)$
- This stationary distribution can be shown to be

$$\boldsymbol{\lambda}^{\intercal} = \left(\frac{\pi_{\phi i}}{\pi_{\ell p} + \pi_{\phi i}} \quad \frac{\pi_{\ell p}}{\pi_{\ell p} + \pi_{\phi i}} \right)$$

Equilibrium Distribution of Latent Mastery

$$P(X_{pi}=1) = P(X_{pi}=1|Z_{pi}=1)P(Z_{pi}=1) + P(X_{pi}=1|Z_{pi}=0)P(Z_{pi}=0)$$
$$= (1-\pi_{si})\frac{\pi_{\ell p}}{\pi_{\ell p}+\pi_{\phi i}} + \pi_{gi}\frac{\pi_{\phi i}}{\pi_{\ell p}+\pi_{\phi i}}$$

let $\theta_p = \log \pi_{\ell p}$ and $\delta_i = \log \pi_{\phi i}$

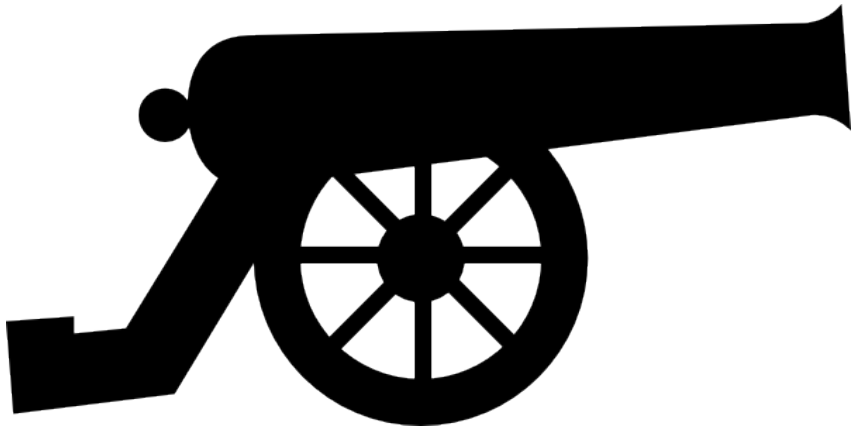
$$= (1-\pi_{si})\frac{\exp(\theta_i-\delta_i)}{1+\exp(\theta_p-\delta_i)} + \pi_{gi}\left(1-\frac{\exp(\theta_p-\delta_i)}{1+\exp(\theta_p-\delta_i)}\right)$$
$$= \pi_{gi} + ((1-\pi_{si})-\pi_{gi})\frac{\exp(\theta_p-\delta_i)}{1+\exp(\theta_p-\delta_i)}$$

The Criticism

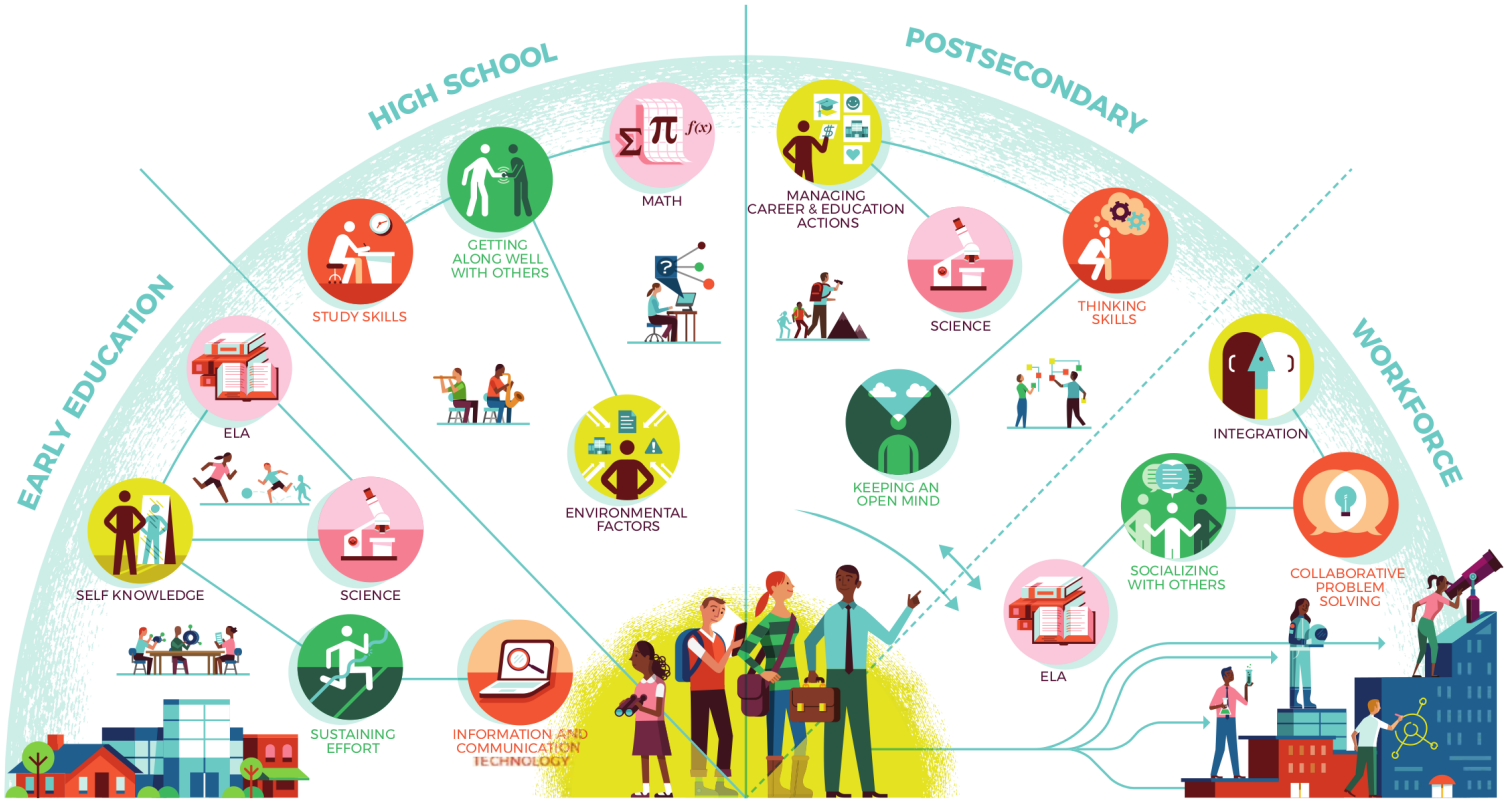
- No placeholder for education
- No component in either model to denote teaching or education that is occurring to the learners
- No component to explain differences in teaching lead to differences in learning outcomes (IRT), or the learning process (BKT).

Ballistic Model

- BKT model is like firing a canon
- initial conditions akin to parameters
- Learning akin to trajectory
- Reviewers argued role of education in this system is to calibrate such a canon



Holistic Model

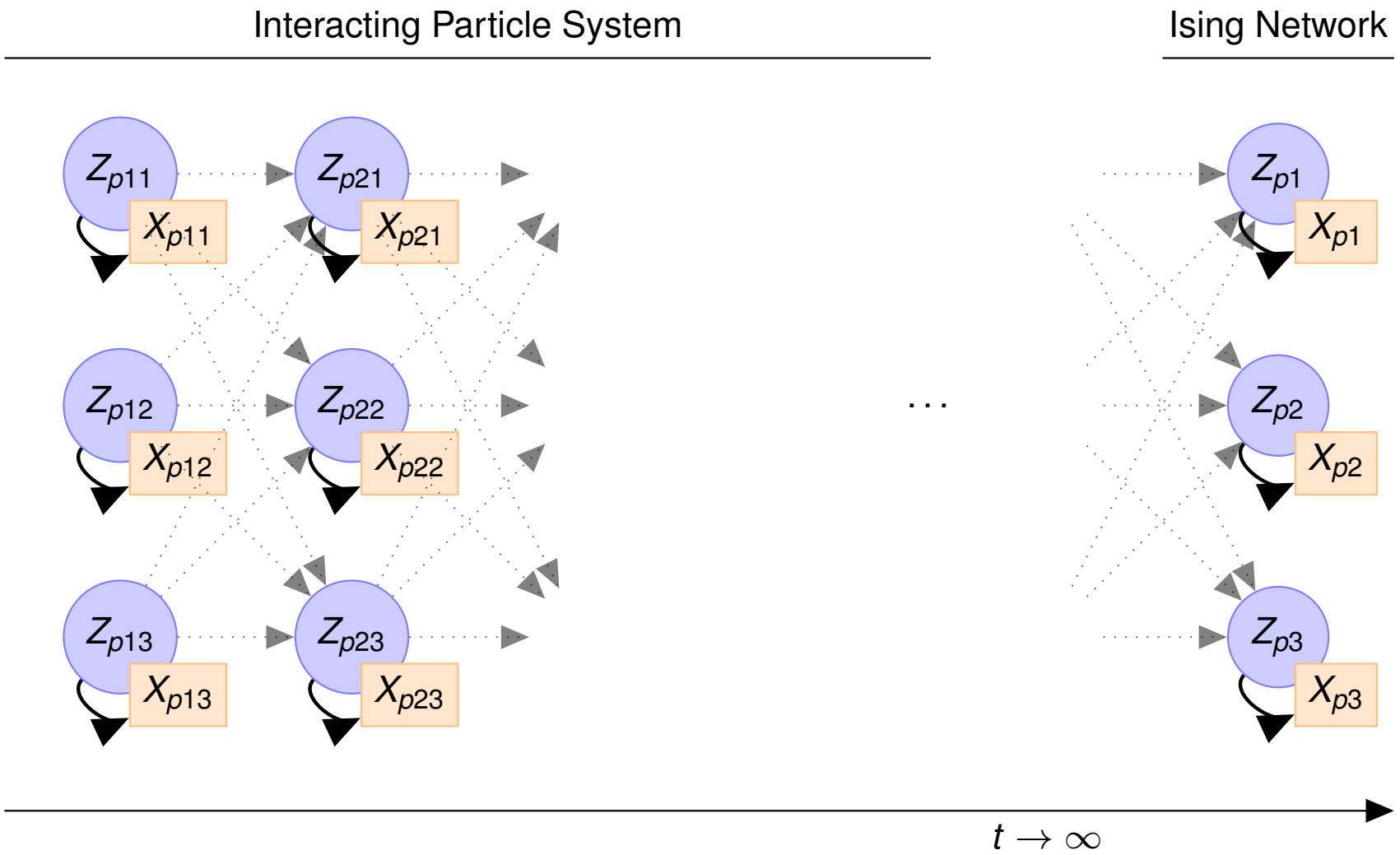


The holistic model of education and work success

Desirable Model Features

- Takes hierarchical dependencies into account
- Take into account well replicated psychological findings
  - positive manifold
  - Matthew effect
  - high predictive validity
- Should express the dynamic and temporal way that students learn/forget/attain skills/knowledge

Connection to Networks



Thank You!

- Collaborators
  - Michael Yudelson
  - Maria Bolsinova
  - Meirav Attali
  - Gunter Maris
- Any Questions?

References

Corbett, A. T. and Anderson, J. R. (1994). “Knowledge tracing: Modeling the acquisition of procedural knowledge”. In: *User modeling and user-adapted interaction* 4.4, pp. 253–278.

Deonovic, B., Yudelson, M., Bolsinova, M., Attali, M., and Maris, G. (2018). “Assessment meets Learning: On the relation between Item Response Theory and Bayesian Knowledge Tracing”. In: *ArXiv e-prints*. arXiv: 1803.05926 [stat.ME].